Advanced Control Algorithms for Satellite Attitude Maneuver

Debajyoti Chakrabarti
SC15B084

B.Tech Porject

Supervisor: Dr. N. Selvaganesan

Indian Institute of Space Science and Technology
Thiruvananthapuram, Kerala, India 695547

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Outline

Outline (contd.)

- Tuning of Parameters

Previous Work

Sl No.	Name of Conference or Journal Paper	Author(s)	Conference or Journal Name	Content
1	Sliding mode control using modified rodrigues parameters	J. L. Crassidis, F. L. Markley (1996)	Journal of Guidance,Control and Dynamics	Robust Sliding Mode Control
2	Systematic design of attitude control systems for a satellite in a circular orbit with guaranteed performance and stability	R. Hull, C. Ham, R. Johnson (2000)	AIAA/USU conference on small satellites	Non linear controller Using Euler angle formulation
3	Performances comparison of linear and sliding mode attitude controllers for flexible spacecraft with reaction wheel	E. Abdulhamitbilal, E. M. Jafarov (2006)	International Workshop on Variable Structure Systems, IEEE	Comparision of Linear controller and SMC
4	Sliding mode attitude tracking of rigid spacecraft with disturbances	Kunfeng Lu, Yuanqing Xia, Zheng Zhu Michael V. Basin (2012)	Journal of Franklin Institute	Sliding mode disturbance observer
5	A fractional order sliding mode controller design for spacecraft attitude control system	Z. Tianyi , R. Xuemei, Z. Yao (2015)	Chinese Control Conference (CCC)	SMC with fractional sliding surface for satellite attitude control
6	Improved sliding mode controller performance through power rate exponential reaching law	Devika, KB, Thomas, Susy (2017)	Second International Conference on Electrical, Computer and Communication Technologies (ICECCT)}	Alleviation of chattering by power rate exponential reaching law (PRERL)

Objective

The main objectives of this project work are following:

- Study of various attitude representation techniques
- Modelling of satellite with reaction wheel as actuator
- Tuning of integer order and fractional order controllers
- Tuning of sliding mode and fractional sliding mode based controllers
- Stability analysis of sliding mode based controllers
- Validation of all the tuned controllers, analysis of disturbance rejection and robustness property

Basic Definitions

- Attitude of a satellite
- Attitude Determination and Control System (ADCS)

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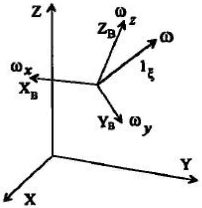

Figure 1: Body frame wrt Reference frame[28]

Attitude Representation Methods

- Direction Cosine Matrix (DCM)
- Euler Angles
- Quaternions

Advantage of Quaternions

- Only four quantities
- Absence of **Gimbal Lock** problem
- Simplicity of dynamics equation

In this work, Quaternion method is used for attitude representation

Mathematical Model of Satellite

Mathematical modelling of reaction wheel actuated satellite consists of:

- Modelling of satellite attitude
- Modelling of reaction wheel assembly

Modelling of satellite attitude consists of:

1. Rotational Dynamics
2. Kinematics

Rotational Dynamics of Satellite

- For dynamics modelling, satellite is taken as Rigid body and point mass model is used to describe the dynamics.
- Dynamics is governed by *Conservation of Angular Momentum*.

$$T_e = \dot{H}_{inertial} \tag{1}$$

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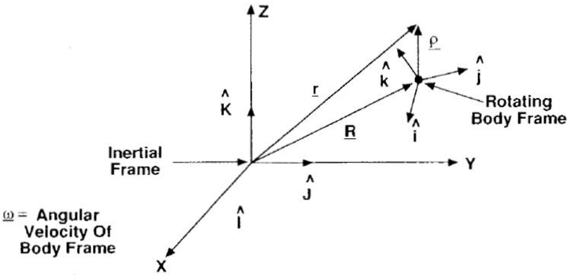

Figure 2: Time Derivative in Rotating frame

Contd.

- From Newtonian mechanics, given a vector ρ,

$$(\frac{d\rho}{dt})_i = (\frac{d\rho}{dt})_b + \omega \times \rho \tag{2}$$

- Hence using eqn.(2),

$$\dot{H}_{inertial} = \dot{H} + \omega \times H \tag{3}$$

here $H_{inertial}$ angular momentum in inertial frame & H, ω are in body frame)

- Moment of inertia tensor of the satellite body be J, then $H_b = J\omega$

Contd.

- If angular momentum of rotation element (reaction wheel assembly here) be ΣH_i then,

$$H = H_b + \Sigma H_i \tag{4}$$

- Now from (1) (2) (3) (4),

$$T_e = \dot{H}_b + \Sigma \dot{H}_i + \omega \times (H_b + \Sigma H_i) \tag{5}$$

This is the general equation describing satellite rotational dynamics.

Reaction Wheel as Actuator

- Reaction wheel assembly is used as actuator in this work.Hence $\Sigma H_i = h_w$ (h_w is the net wheel angular momentum along sbc). Therefore,

$$-\dot{h}_w = -\Sigma \dot{H}_i = T_c \quad (6)$$

T_c is the control torque on satellite. Now eqn.(5) simplifies as,

$$\dot{H}_b = T_e + T_c - \omega \times (H_b + h_w) \quad (7)$$

This is the complete dynamics of satellite with reaction wheel as actuator.

- Control torque due to wheel comes as internal torque.
- External torque(T_e) in (7) is only the disturbance acting on satellite.

Kinematics of Satellite

- **Kinematics** of satellite is defined in terms of quaternions. Quaternion rate eqn is given by:

$$\dot{q} = \frac{1}{2} q \otimes \omega \tag{8}$$

  ($\otimes$ means quaternion multiplication)

- As $H_b = J\omega$, So integration of eqn. (7) gives H_b, which finally gives ω in body frame, which can be used to propagate quaternion rate eqn. (8) and find $q(t)$ of next time instant.

Actuator Dynamics

- T_w be the torque from stator to the rotor of reaction wheel. Dynamic relation between T_w and T_c describing the complete actuator dynamics is as below,

$$
\begin{aligned}
\tau_w \ddot{h}_w &= -\dot{h}_w + T_w \\
T_c &= -\dot{h}_w \quad \text{(from eqn.(6))} \\
h_w(t) &= \int_0^t \dot{h}_w(\tau) d\tau + h_w(0)
\end{aligned}
\tag{9}
$$

where τ_w is the time constant of the reaction wheel.

- In laplace domain, the relation between T_c and T_w is given by,

$$
T_c = -\frac{1}{\tau_w s + 1} T_w \tag{10}
$$

Combined Nonlinear Model

- From the equations (7),(8),(9), overall nonlinear model for the angular motion of the satellite is given by,

$$\frac{d}{dt}\begin{bmatrix} \omega \\ q \\ q_4 \\ h_w \\ \dot{h}_w \end{bmatrix} = \begin{bmatrix} J^{-1}(-\omega \times (J\omega + h_w) + T_c + T_d) \\ -\frac{1}{2}\omega \times q + \frac{1}{2} q_4 I_{3\times 3}\omega \\ \\ -\frac{1}{2}\omega^T q \\ -T_c \\ -\frac{1}{\tau_w}\dot{h}_w + \frac{1}{\tau_w} T_w \end{bmatrix} = f \quad (11)$$

Openloop Dynamics

Openloop Dynamics

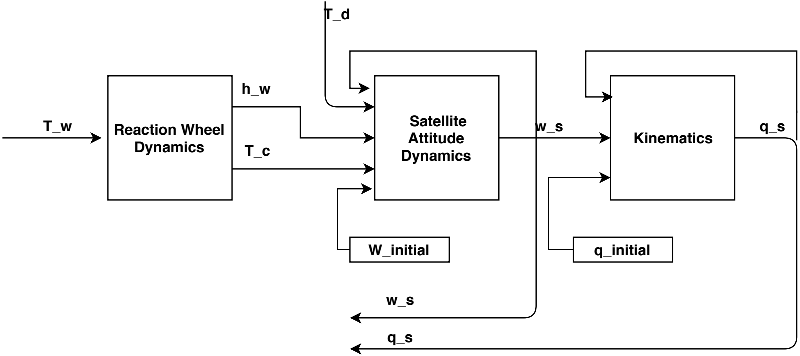

Figure 3: Open Loop Dynamics of Satellite

Openloop Dynamics

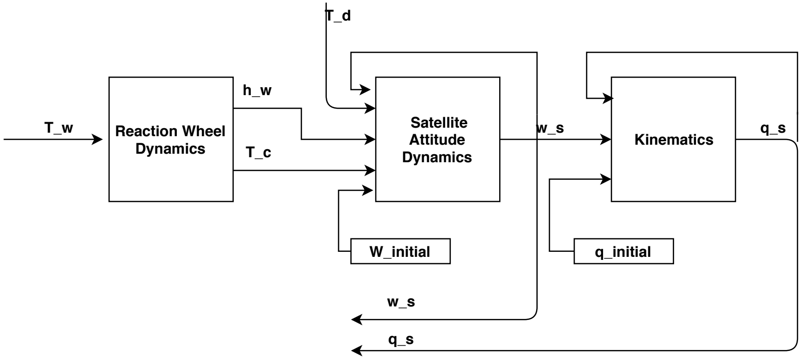

Figure 3: Open Loop Dynamics of Satellite

- Fig. 3 shows open loop dynamics of satellite
- h_w be the angular momentum of reaction wheel
- T_c control torque & T_d be the disturbance to satellite q_s, ω_s are measured angular velocity and quaternion of satellite

Linearization of Satellite Model

- For linearization of the non linear model, state vector x is defined as, $x = [\omega, q, h_w, \dot{h}_w]^T$
- The steady state point of operation be denoted by $\bar{x} = [\bar{\omega}, \bar{q}, \bar{h_w}, \bar{\dot{h_w}}]^T$ and deviation from operating point be denote by, $\Delta x = [\Delta\omega, \Delta q, \Delta h_w, \Delta \dot{h}_w]^T$

- The state space model of linearized satellite model[] is given as below,

$$\Delta\dot{x}(t) = A(t)\Delta x(t) + B_u(t)\Delta T_w(t) + B_d \Delta T_d(t) \tag{12}$$

where jacobian matrices $A_{ij} = \frac{\partial f_i}{\partial x_j}$ and $B_{ij} = \frac{\partial f_i}{\partial u_j}$ (here u being T_w which equals to $[T_{w1}, T_{w2}, T_{w3}]^T$)

In this work, operating point is chosen as, $\bar{q} = [0, 0, 0, 1]^T$; $om\bar{e}ga = [0.02, -0.07, 0.05]^T$; $\bar{h}_w = [0, 0, 0]^T$; $\bar{\dot{h}}_w = [0, 0, 0]^T$

Design of Attitude Controller

- The domain of attitude control can be divided into two parts:

1. **Passive Attitude Control**
   $\Longrightarrow$ Spin Stabilization
   $\Longrightarrow$ Gravity-Gradient Stabilization
   $\Longrightarrow$ Aerodynamic & Solar Pressure Stabilization
2. **Active Attitude Control**

- Considering several flaws of passive control, Three axis active attitude control has been considered in this work.
- Under active attitude controllers, two classes of controllers have been designed in this work namely,

1. **Linear Controller**
2. **Non Linear Controller**

Linear Controller

- Two types of linear controllers are considered:

1. *PD* Controller
2. PD^{β} Controller

- Optimization method is used for tuning of the controllers
- Control effort ($\|\vec{u}(t)\|_2^2$) minimization has been considered as objective
- Following frequency domain specifications are considered as constraints while designing the controller,

- **Phase Margin Specification**

$$Arg[G(j\omega_{gc})G_c(j\omega_{gc})] + \pi < \phi_m \tag{13}$$

where ϕ_m is the specified upper limit of the required phase margin, ω_{gc} is the required gain crossover frequency.
G, G_c be the plant and controller transfer function.

- Other specifications considered are Gain crossover frequency specification & Isodamping specification (also known as ‘robustness towards gain variation’).

- **Gain Crossover Frequency Specification**
  At gain crossover frequency ω_{gc}, open loop transfer function $G(s)G_c(s)$ satisfies following condition,

$$|G(j\omega_{gc})G_c(j\omega_{gc})| = 1 \tag{14}$$

- **Isodamping Specification**

$$\left.\frac{d(Arg[G(j\omega)G_c(j\omega)])}{d\omega}\right|_{\omega=\omega_{gc}} = 0 \tag{15}$$

- Depending on controller structure, number of specifications to be used is decided.

Generalized Block Diagram Representation

Generalized Block Diagram Representation

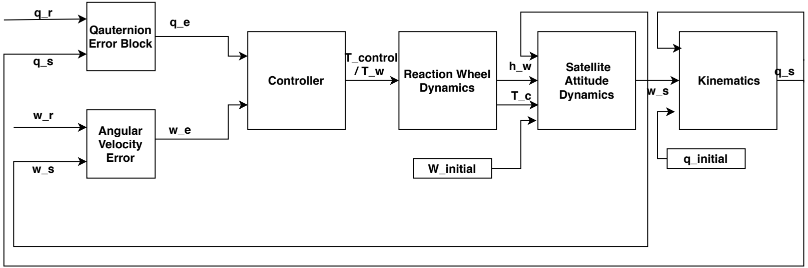

Figure 4: Block Diagram of Linear Attitude Control System

Generalized Block Diagram Representation

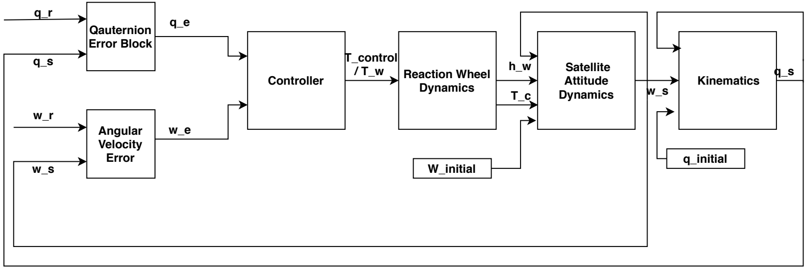

Figure 4: Block Diagram of Linear Attitude Control System

- error quaternion (q_{er}) is defined as,

$$q_{er} = q_s^{-1} \otimes q_r \quad (q_r \ \& \ q_s \text{ be ref. \& measured quaternions}) \tag{16}$$

$$\implies q_e = [q_{er1}, q_{er2}, q_{er3}]^T, \ \omega_e = \omega_r - \omega_s$$

ω_r be reference angular velocity, ω_s measured angular velocity

PD Controller

- The PD controller structure is considered as,

$$G_{ci}(s) = K_{pi}(1 + K_{di}s) \quad \forall i = 1(.)3 \tag{17}$$

  - Phase margin and gain crossover frequency conditions are chosen as optimization constraints.
  - Consider, $G_i(s)$ be the ith axis direct transfer function. Then the optimization problem for PD controller design can be summarized as,

$$\begin{aligned} \text{minimize} \quad & \|\vec{u}(t)\|_2^2 \\ \text{subject to} \quad & K_{di}\,\omega_{gc} - \tan(A) < 0 \\ & |G_i(j\omega_{gc})|K_{pi}\sqrt{1 + (K_{di}\,\omega_{gc})^2} - 1 = 0 \end{aligned}$$

where $A = \phi_m - \pi - \angle G_i\Big|_{\omega_{gc}}$

  - In this work, ω_{gc} and ϕ_m are set as follows[39] ,

$$\omega_{gc} = 1\ rad/s \text{ and } \phi_m = 80^\circ$$

Contd..

- **Control Law Design**:
  Gain values are separately designed for three axes, by solving optimization problem

- PD control law formulated for non linear satellite model is as follows,

$$T_{control} = 2K_P\, q_e + K_D\, \omega_e \tag{18}$$

where $K_P = \begin{bmatrix} K_{p1} & 0 & 0 \\ 0 & K_{p2} & 0 \\ 0 & 0 & K_{p3} \end{bmatrix}$,

$$K_D = \begin{bmatrix} K_{p1}K_{d1} & 0 & 0 \\ 0 & K_{p2}K_{d2} & 0 \\ 0 & 0 & K_{p3}K_{d3} \end{bmatrix}$$

PD^{β} Controller

- Structure of PD^{β} is considered as,

$$G_{cfi}(s) = K_{pi}(1 + K_{di} s^{\beta_i}) \quad \forall i = 1(.)3 \tag{19}$$

  - $s^{\beta} \implies$ Caputo fractional operator of order β
  - Phase margin, gain cross over frequency, Isodamping condition are chosen as optimization constraints
  - Formulation of Optimization is as below,

minimize $\|\vec{u}(t)\|_2^2$
subject to

$$K_{di}\, \omega_{gc}^{\beta_i} + \sin\frac{(1-\beta_i)\pi}{2} - \cos\frac{(1-\beta_i)\pi}{2}\tan(B) < 0$$

$$|G_i(j\omega_{gc})| K_{pi} \sqrt{(1 + K_{di}\, \omega_{gc}^{\beta_i} \cos\frac{\beta_i \pi}{2})^2 + (K_{di}\, \omega_{gc}^{\beta_i} \sin\frac{\beta_i \pi}{2})^2} - 1 = 0$$

$$\left.\frac{d\angle G_i}{d\omega}\right|_{\omega_{gc}} + \frac{K_{di}\ \beta_i\ \omega_{gc}^{\beta_i - 1}}{\cos\frac{(1-\beta_i)\pi}{2}(1 + \tan^2 B)} = 0$$

Cont..

- where $B = \frac{(1-\beta_i)\pi}{2} + A$
- **Control Law Design:**
  Optimization problem posed above is solved for the parameters of PD^{β} controller

- PD^{β} law for satellite model is formulated as,

$$T_{control} = 2K_P\ q_e + K_D\ D^{\beta-1}\omega_e \tag{20}$$

K_P, K_D definitions same as the definitions given for PD controller

Non Linear Controller

- Main objective is to design nonlinear class of controller for better robustness and disturbance rejection property

- Under the class of non linear controllers, four types of controllers are designed:
  1. *PD* Sliding Mode Controller (PD-SMC)
  2. *PD* Sliding Mode Controller with Modified Reaching Law
  3. PD^{β} Sliding Mode Controller (PD-SMC)
  4. PD^{β} Sliding Mode Controller with Modified Reaching Law

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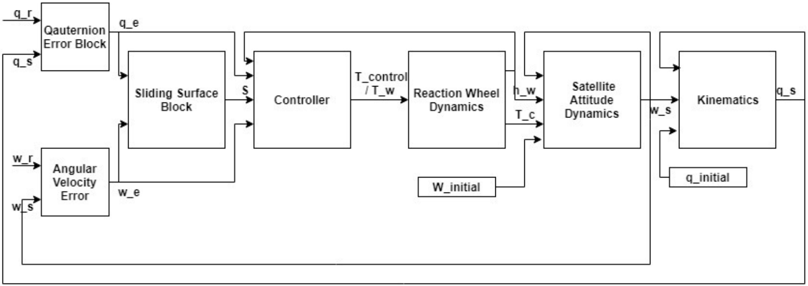

Figure 5: Block Diagram of Non Linear Attitude Control System

Generalized Block Diagram Representation

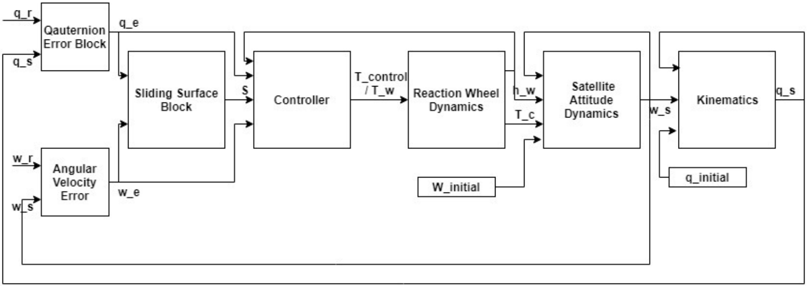

Figure 5: Block Diagram of Non Linear Attitude Control System

- Block diagram almost same as linear controller case
- Only difference, error in attitude and angular velocity provided to sliding surface block

PD-SMC Controller

PD-SMC Controller

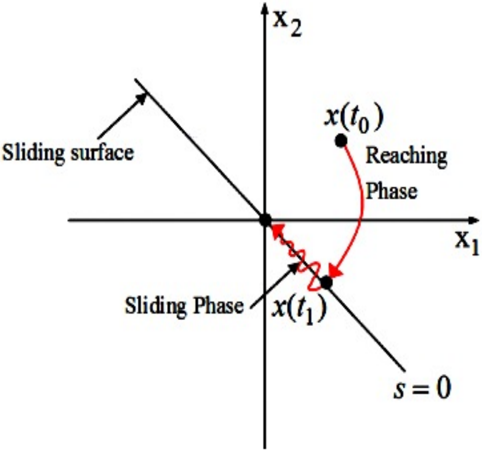

Figure 6: Sliding surface with sliding phases

Contd..

- Design of PD-Sliding mode Control consists of two phases namely,
  1. Design of sliding surface
  2. Determination of control law that stabilizes the system
- The main objective to chose control law such that both attitude error(q_e) & angular velocity error(ω_e) attains zero

Sliding Surface Design:
PD control law The sliding variable design,

$$S = 2K_P q_e + K_D \omega_e \tag{21}$$

K_P, K_D are parameters to be tuned

Contd..

- **Control Law Design**:
From theory of sliding mode, control law has two components namely, *Equivalent Control & Discontinuous Control* i.e.

$$u(t) = u_{eq}(t) + u_{discont}(t) \tag{22}$$

- **Equivalent Control Law Design**:
When system reaches sliding surface($\dot{S} = 0$), equivalent control $u_{eq}(t)$ only works
From $\dot{S} = 0$, expression of $u_{eq}(t)$ is given as,

$$u_{eq}(t) = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} JQ_e\omega_e \tag{23}$$

Contd..

- where the following relation is used:

$$\dot{q}_{er} = \begin{bmatrix} \dot{q}_e \\ \dot{q}_{e4} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_{e4}\omega_e - \omega_e \times q_e \\ -\omega_e^T q_e \end{bmatrix} \qquad (24)$$

therefore relation between $\dot{q}_e$ and ω_e can be written as,

$$\begin{aligned} \dot{q}_e &= \frac{1}{2}(q_{e4}\,\omega_e - \omega_e \times q_e) \\ &= \frac{1}{2}\begin{bmatrix} q_{e4} & -q_{e3} & q_{e2} \\ q_{e3} & q_{e4} & -q_{e1} \\ -q_{e2} & q_{e1} & q_{e4} \end{bmatrix} \omega_e \qquad (25) \\ &= \frac{1}{2} Q_e \omega_e \end{aligned}$$

Contd..

- **Discontinuous Control Law Design**:
  Discontinuous control law is chosen as,

$$u_{discont}(t) = -K\ sgn(S) \tag{26}$$

  where K is a scalar to be tuned

Hence expression of net control torque for $PD - SMC$ is given by,

$$T_{control} = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} JQ_e\omega_e - Ksgn(S) \tag{27}$$

- **Proof of Stability,Convergence & Calculation of Reaching Time**: proofs are discussed in thesis
- **Tuning of Parameters**:
  Tuning of parameters K_P, K_D, K are discussed later
- The main disadvantage of *PD*-SMC is: **Chattering Effect**

- Different ways to remove ‘chattering effect’, one option is **‘boundary layer control’**
- Replacement of saturation by signum func. which is given by,

$$sat(S,\phi) = \begin{cases} 1, & for \;\; S > \phi \\ \frac{S}{\phi}, & for \;\; |S| \leq \phi \\ -1, & for \;\; S < \phi \end{cases} \tag{28}$$

where ϕ is the thickness of boundary layer, S be the sliding surface.

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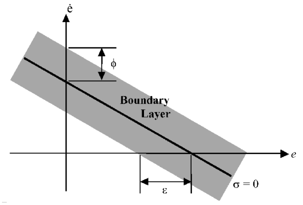

Figure 7: Effect of Saturation Function **[57]**

- Disadvantage of this method, Reduction in robustness
- States confined in the boundary region
- $\phi \rightarrow 0$, ideal sliding mode occurs

PD-SMC with Modified Reaching Law

- Another method to remove chattering: Here **Power rate reaching law**

- Reaching law used for controller design,is given by,

$$\dot{S} = -K_m|S|^{\alpha} sgn(S) \tag{29}$$

where K_m is positive diagonal matrix and $0 < \alpha < 1$,
$|S|^{\alpha} = diag\{|S_x|^{\alpha}, |S_y|^{\alpha}, |S_z|^{\alpha}\}$

- Advantage of this method: chattering control with a control over robustness
- Control of chattering and robustness, both depend on choice of α(discussed later)

Contd..

Choice of α:

power coefficient α is chosen by analyzing following plot

Contd..

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power coefficient α is chosen by analyzing following plot

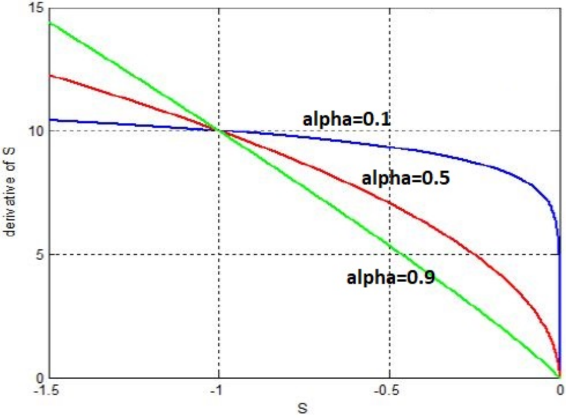

Figure 8: Variation of derivative of S with S **[57]**

- α controls convergence rate
- α near 1 can even skip the sliding phase $\Longrightarrow$ loss of robustness
- In this work, α chosen to be 0.5

Contd..

- **Sliding Surface Design**:
  Sliding surface chosen here is same as in case of $PD - SMC$ (21)
- Control Law derived directly by comparing $\dot{S}$ with the reaching law (29) .
- **Design of Control Law**:
  From eqn (21), $\dot{S} = 2K_P\,\dot{q}_e + K_D\,\dot{\omega}_e$
  Comparison of $\dot{S}$ with (29) gives,

$$T_{control} = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} JQ_e\omega_e - \frac{K_m}{K_D} J|S|^{\alpha} sgn(S) \quad (30)$$

- **Proof of Stability,Convergence & Calculation of Reaching Time**: Proofs are discussed in thesis
- **Tuning of Parameters**:
  Tuning of parameters K_m with K_P, K_D is discussed later

PD^{β}-SMC

- Design procedure of $PD^{\beta} - SMC$, similar to $PD - SMC$
- Two steps for controller design: Sliding manifold design, suitable control law design based upon manifold

- **Sliding Surface Design**:
Sliding surface suggested in this work is as follows,

$$S = 2K_P q_e + K_D D^{\beta}\, \omega_e \tag{31}$$

K_P,K_D need to be tuned

- **Control Law Design**:
- Design of control law consists of $u_{eq}(t)$ & $u_{discont}(t)$, discussed next

Contd..

- **Equivalent Control Law Design:** Comparing $\dot{S}$ to zero, expression for u_{eq} is obtained as,

$$u_{eq} = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} J D^{-\beta} Q_e \omega_e \tag{32}$$

- **Discontinuous Control Law Design:**

$$u_{discont} = -K_f D^{-\beta} sgn(S) \tag{33}$$

Expression of net control torque in case of PD^{β}-SMC controller,

$$T_{control} = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} J D^{-\beta} Q_e \omega_e - K_f D^{-\beta} sgn(S) \tag{34}$$

- **Proof of Stability,Convergence & Calculation of Reaching Time**: Proofs are discussed in thesis
- **Tuning of Parameters:** Tuning of parameters K_P, K_D, β, K_f is discussed later

PD^{β}-SMC with Modified Reaching Law

- Reaching law of '$PD^{\beta} - SMC$ with Modified Reaching Law' controller is similar to (29) i.e. Power rate reaching law

$$\dot{S} = -K_{mf}|S|^{\alpha} sgn(S) \tag{35}$$

- **Sliding Surface Design**:
  Sliding surface chosen here same as in case of $PD^{\beta} - SMC$ (31)
- **Choice of α**:
  Method to choose α is explained for 'PD-SMC with Modified Reaching Law'.
- **Tuning of Parameters:**
  Design of $\boldsymbol{K_P}, \boldsymbol{K_D}, \boldsymbol{K_{mf}}$ is discussed later

Contd..

- Control Law is derived in similar way of modified PD-SMC
- **Control Law Design:**
  From eqn (31), $\dot{S} = 2K_P\, \dot{q}_e + K_D D^{\beta}\, \dot{\omega}_e$

Comparing $\dot{S}$ with (29) expression of net control torque simplifies to,

$$T_{control} = \omega_e \times (J\omega_e + h_w) - \frac{K_P}{K_D} J D^{-\beta} Q_e \omega_e - \frac{K_{mf}}{K_D} J D^{-\beta} (|S|^{\alpha} sgn(S)) \quad (36)$$

- **Proof of Stability,Convergence & Calculation of Reaching Time**:
  All proofs with derivation of reaching time, are discussed in thesis

Tuning of Parameters

- Each non linear control law has few parameters(x) to be tuned
- In case of $PD - SMC$ control law $x = [K_P, K_D, K]$
- $PD^\beta - SMC$ has 4 parameters to be tuned namely, $x = [K_P, K_D, K_f, \beta]$
- For '$PD - SMC$ with modified Reaching Law' & '$PD^\beta - SMC$ with modified Reaching Law' x is $[K_P, K_D, K_m]$ & $[K_P, K_D, K_{mf}]$ respectively
- Formulation of optimization problem for tuning is,

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & C_0 \sum_{i=1}^{3} \int_t t|\omega_{ei}(x,t)|dt + C_1 \sum_{i=1}^{3} \int_t T^2_{control,i}(x,t)dt \\ & + C_2 \sum_{i=1}^{3} \int_t \omega_{ei}^2(x,t)dt + C_3 \sum_{i=1}^{3} \int_t t|q_{ei}(x,t)|dt \end{aligned} \tag{37}$$

- In this work, $C_x = [0.31; 0.15; 0.18; 0.36]^T$

Simulation Results

- The parameters and its values of the satellite used for simulation is listed in the Table 1.

Parameters	**Value**
Initial Quaternion	$[0.5 \quad 0.5 \quad 0.5 \quad 0.5]$
Initial Angular Velocity (in rad/sec)	$[0.02 \quad -0.07 \quad 0.05]$
Desired Quaternion	$[0 \quad 0 \quad 0 \quad 1]$
Desired Angular Velocity (in rad/sec)	$[0 \quad 0 \quad 0]$
Inertia Matrix of the Satellite	$diag[50, 75, 100]$
Inertia Matrix of the Reaction Wheel	$0.7259 \times 10^{-4} I_{3\times3}$
Maximum Torque of the Reaction wheel	$1N-m$
Time Constant of the Reaction Wheel	$100ms$

Table 1: Parameters of the Satellite

PD Controller

- Designed gain values for PD controller using optimization problem formulated, is given by:
  $K_P = diag\{16.6086; 20.9914; 43.3096\}$ and
  $K_D = diag\{99.0545; 149.493; 196.867\}$

PD Controller

- Designed gain values for PD controller using optimization problem formulated, is given by:
  $K_P = diag\{16.6086; 20.9914; 43.3096\}$ and
  $K_D = diag\{99.0545; 149.493; 196.867\}$

Case 1: *Without Disturbance*

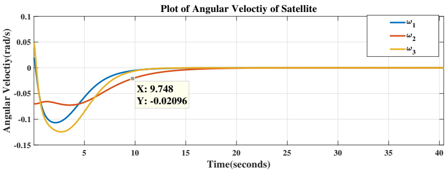

Figure 9: Angular Velocity Response

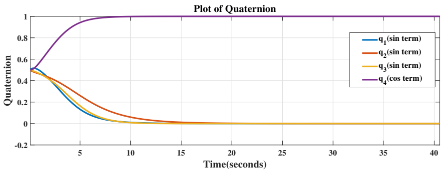

Figure 10: Quaternion Response

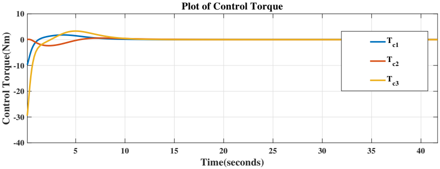

Figure 11: Control Torque Response

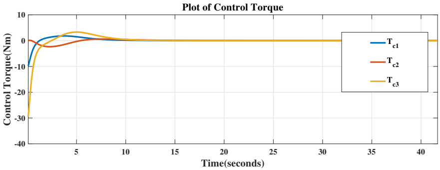

Figure 11: Control Torque Response

Case 2: *With Disturbance*: Random noise applied as disturbance between from $t = 60$ sec to $t = 120$ sec

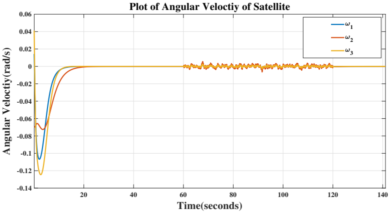

Figure 12: Angular Velocity Response with Disturbance

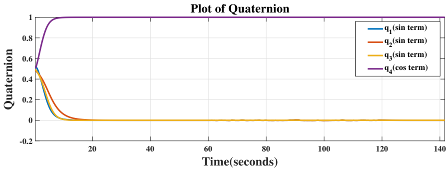

Figure 13: Quaternion Response with Disturbance

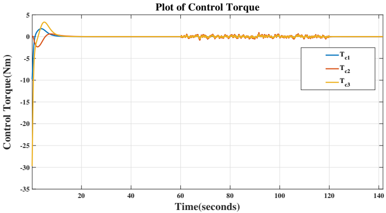

Figure 14: Torque in presence of Disturbance

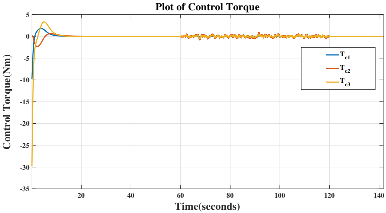

Figure 14: Torque in presence of Disturbance

Case 3: *With Torque Saturation:*

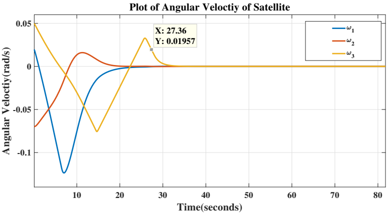

Figure 15: Angular Velocity Response with Saturation

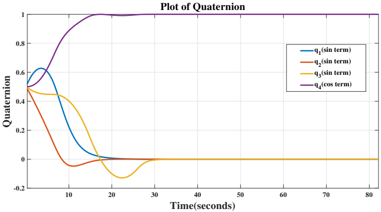

Figure 16: Quaternion Response with Saturation

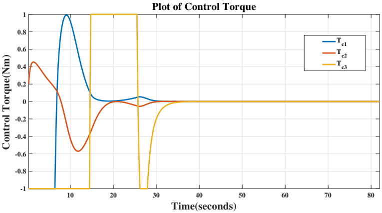

Figure 17: Torque with Saturation

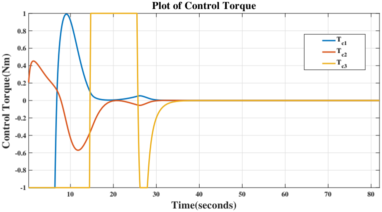

Figure 17: Torque with Saturation

Case 4: *With Perturbation*: Inertia tensor J is perturbed by 40% & 60% of its nominal value as given in table (1), results are shown for 60% pert

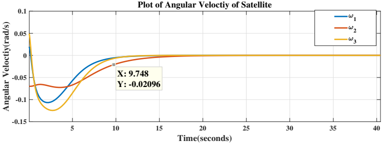

Figure 18: Angular Velocity with J Perturbation

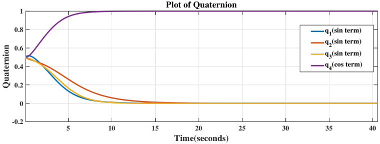

Figure 19: Quaternion with J Perturbation

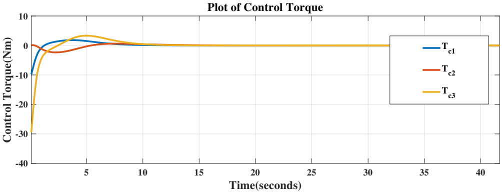

Figure 20: Control Torque with J perturbation

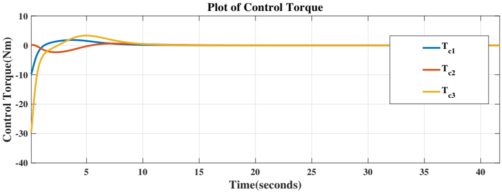

Figure 20: Control Torque with J perturbation

Observations

From the above simulation results it can be observed that:

- Using PD controller, system is able to track desired attitude with settling time $T_s = 9.748$ sec
- System is able to reject disturbance
- System is invariant to J matrix perturbation
- System shows stable response when control torque is saturated to 1 N-m but increment in T_s to 27.36 sec.

PD^β Controller

- Designed gain values for PD controller using optimization problem formulated, is given by:
  $K_P = diag\{16.608; 20.99; 43.309\}$,
  $K_D = diag\{99.05; 149.5; 196.8\}$ &
  $\beta = [0.7439; 0.8120; 0.6632]$

PD^{β} Controller

- Designed gain values for PD controller using optimization problem formulated, is given by:
  $K_P = diag\{16.608; 20.99; 43.309\}$,
  $K_D = diag\{99.05; 149.5; 196.8\}$ &
  $\beta = [0.7439; 0.8120; 0.6632]$

Case 1: *Without Disturbance*

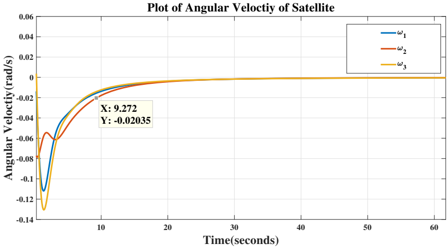

Figure 21: Angular Velocity Response

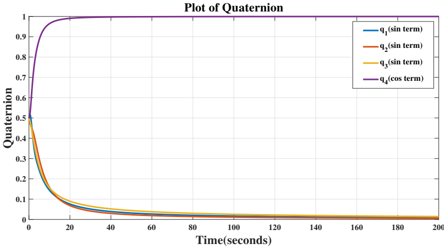

Figure 22: Quaternion Response

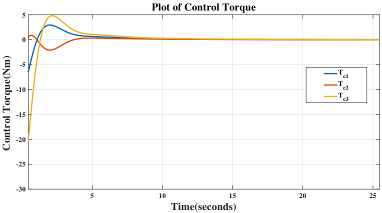

Figure 23: Control Torque Response

Case 2:

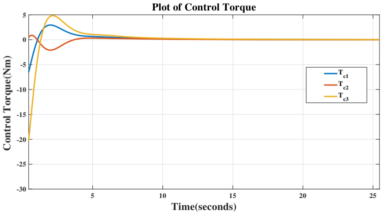

Figure 23: Control Torque Response

Case 2: *With Disturbance*: Random noise applied as disturbance between from $t = 60$ sec to $t = 120$ sec

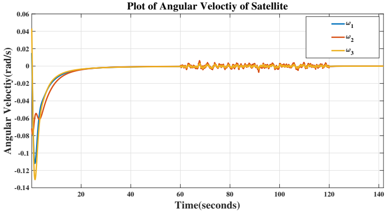

Figure 24: Angular Velocity Response with Disturbance

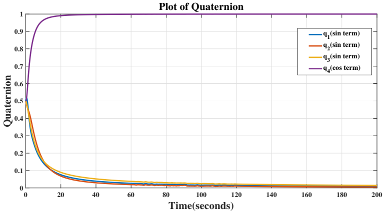

Figure 25: Quaternion Response with Disturbance

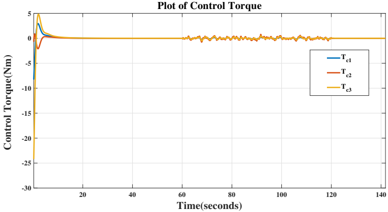

Figure 26: Torque in presence of Disturbance

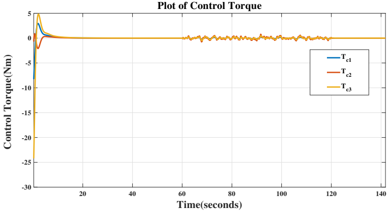

Figure 26: Torque in presence of Disturbance

Case 3: *With Torque Saturation:*

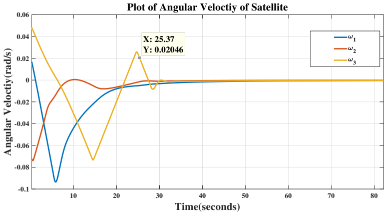

Figure 27: Angular Velocity Response with Saturation

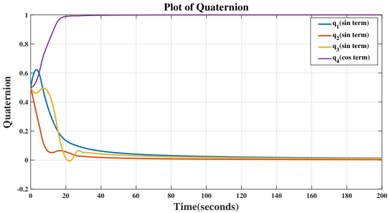

Figure 28: Quaternion Response with Saturation

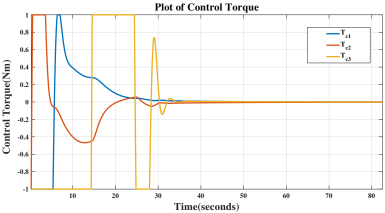

Figure 29: Torque with Saturation

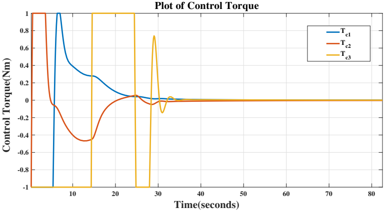

Figure 29: Torque with Saturation

Case 4: *With Perturbation*: Inertia tensor J is perturbed by 40% & 60% of its nominal value as given in table (1), results are shown for 60% pert

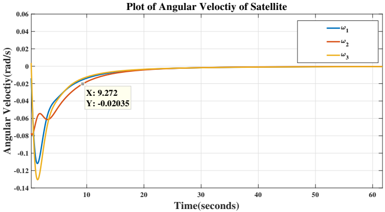

Figure 30: Angular Velocity with J Perturbation

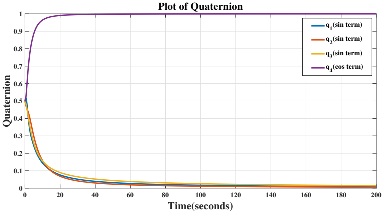

Figure 31: Quaternion with J Perturbation

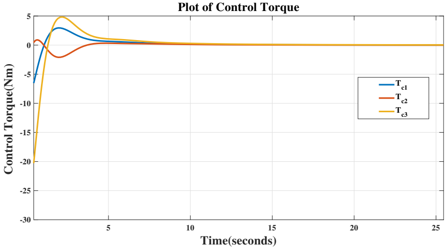

Figure 32: Control Torque with J perturbation

Observations

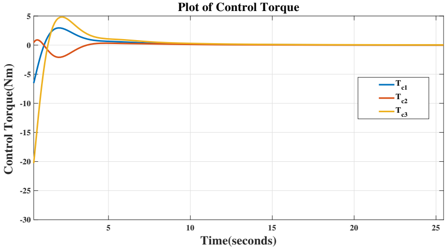

Figure 32: Control Torque with J perturbation

Observations

From the above simulation results it can be observed that:

- Using PD^{β} controller system is able to track desired attitude both in presence and absence of disturbance with $T_s = 9.272$ sec
- System with PD^{β} controller is invariant to J matrix perturbation
- For torque saturation increment in T_s is to 25.37 sec.

Non linear Controller

- Optimization result for non linear controllers is listed below:

Controller	Parameters of the Controller			
	K_P	K_D	β	Gain (K or K_m or K_f or K_{mf})
PD-SMC	2.3997	12.452	None	0.72167
PD-SMC modified	2.5338	10.3046	None	0.21542
PD^{β}-SMC	2.3811	22.6834	0.001	0.6837
PD^{β}-SMC modified	2.2807	25.074	0.001	0.5374

Table 2: Optimized parameters of non linear Controllers

PD-SMC

PD-SMC

Case 1: *Without Disturbance*

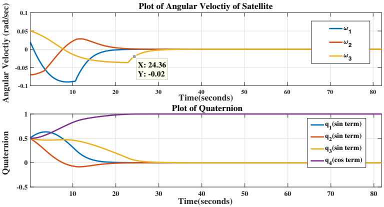

Figure 33: Angular Velocity & Quaternion Response

PD-SMC

Case 1: *Without Disturbance*

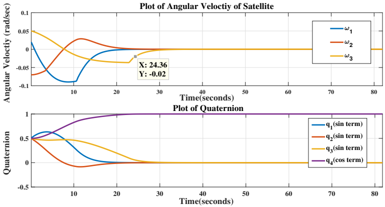

Figure 33: Angular Velocity & Quaternion Response

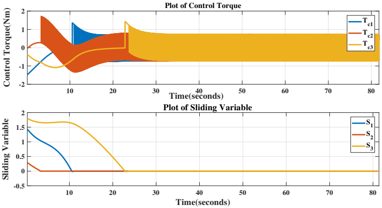

Figure 34: Torque & Sliding Variable Response

Case 2: *With Disturbance*: between 60 to 120 sec

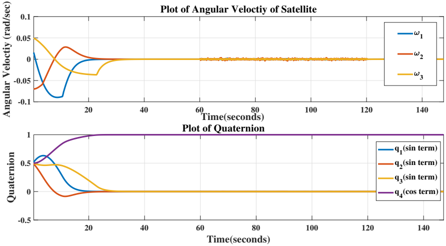

Figure 35: Angular Velocity & Quaternion Response

Case 2: *With Disturbance*: between 60 to 120 sec

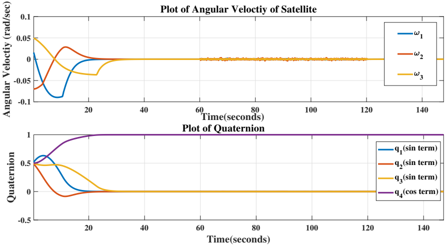

Figure 35: Angular Velocity & Quaternion Response

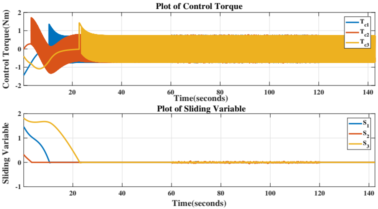

Figure 36: Torque & Sliding Variable Response

Case 3: *With Saturation Function*

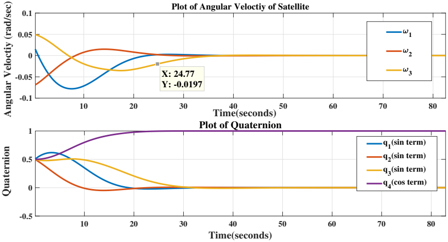

Figure 37: Angular Velocity & Quaternion Response

Case 3: *With Saturation Function*

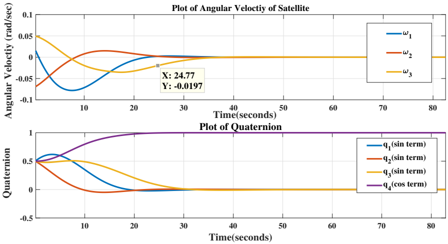

Figure 37: Angular Velocity & Quaternion Response

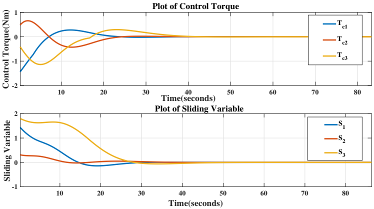

Figure 38: Torque & Sliding Variable Response

Case 4: *With Torque Saturation*

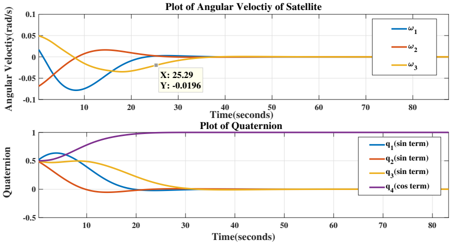

Figure 39: Angular Velocity & Quaternion Response

Case 4: *With Torque Saturation*

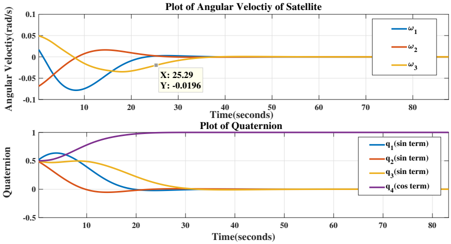

Figure 39: Angular Velocity & Quaternion Response

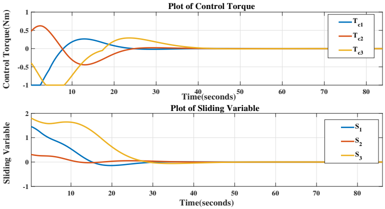

Figure 40: Torque & Sliding Variable Response

Case 5: *With Perturbation*

Both 40% & 60% perturbation applied. Fig. shows reponse for 60% perturbation

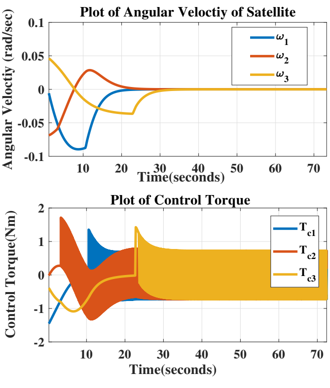

Figure 41: Response in presence of 60 % perturbation

Observation

- For PD-SMC law, following things can be observed:
- System achieves its desired angular velocity & quaternion with T_s=24.36 sec
- Disturbance effect is rejected
- Chattering phenomena can be viewed for PD-SMC, which gets removed using saturation function
- With torque saturation, control torque is limited to ± 1 N-m but T_s raises to 25.29 seconds

PD-SMC with Modified Reaching Law

PD-SMC with Modified Reaching Law

Case 1: *Without Disturbance*

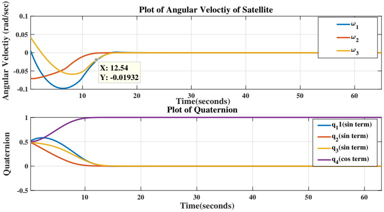

Figure 42: Angular Velocity & Quaternion Response

PD-SMC with Modified Reaching Law

Case 1: *Without Disturbance*

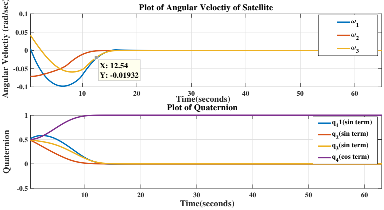

Figure 42: Angular Velocity & Quaternion Response

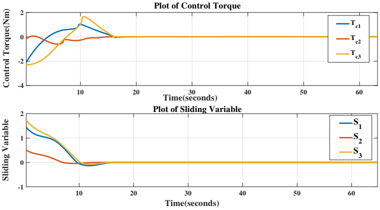

Figure 43: Torque & Sliding Variable Response

Case 2: *Without Disturbance*: between 60 to 120 sec

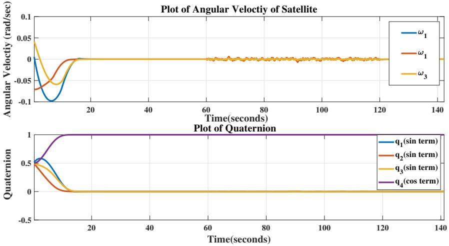

Figure 44: Angular Velocity & Quaternion Response

Case 2: *Without Disturbance*: between 60 to 120 sec

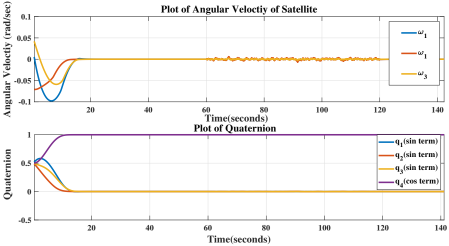

Figure 44: Angular Velocity & Quaternion Response

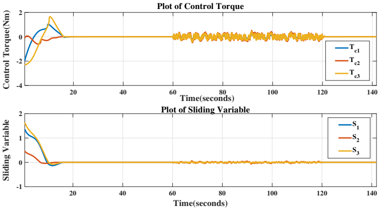

Figure 45: Torque & Sliding Variable Response

Case 3: *With Torque Saturation*

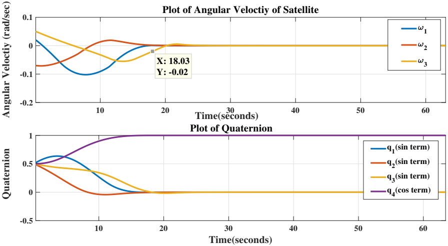

Figure 46: Angular Velocity & Quaternion Response

Case 3: *With Torque Saturation*

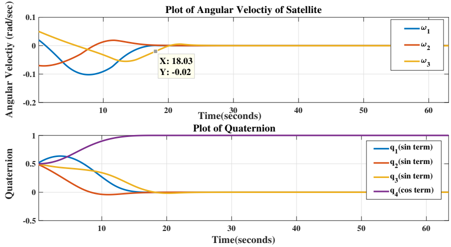

Figure 46: Angular Velocity & Quaternion Response

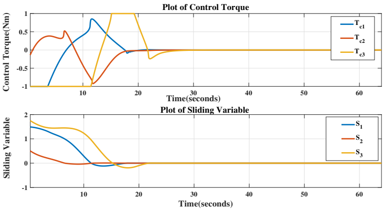

Figure 47: Torque & Sliding Variable Response

Observation

- **Observations**:
- In this case following observations can be mentioned:
- System achieves desired responses with T_s=12.54 sec
- Disturbance effect is rejected
- Chattering effect is absent here
- Limiting the maximum control torque, T_s raises to 18.03 sec

PD^{β}-SMC

PD^{β}-SMC

Case 1: *Without Disturbance*

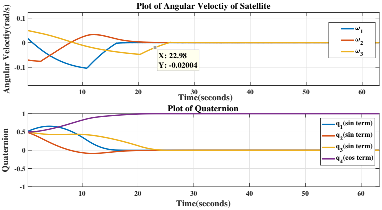

Figure 48: Angular Velocity & Quaternion Response

PD^{β}-SMC

Case 1: *Without Disturbance*

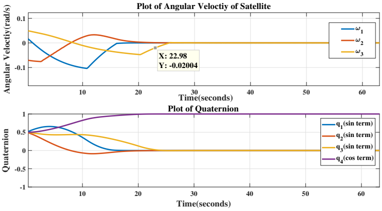

Figure 48: Angular Velocity & Quaternion Response

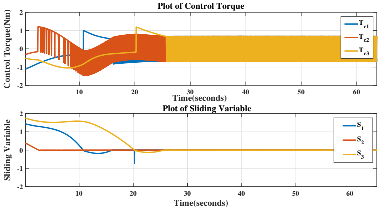

Figure 49: Torque & Sliding Variable Response

Case 2: *With Disturbance*: between 60 to 120 sec

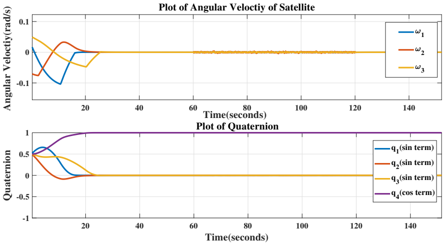

Figure 50: Angular Velocity & Quaternion Response

Case 2: *With Disturbance*: between 60 to 120 sec

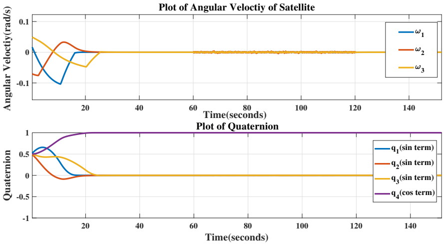

Figure 50: Angular Velocity & Quaternion Response

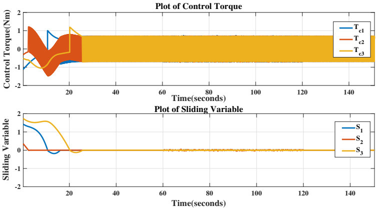

Figure 51: Torque & Sliding Variable Response

Case 3: *With Saturation Function*

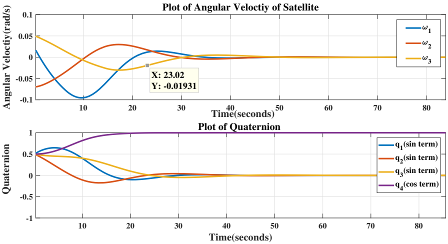

Figure 52: Angular Velocity & Quaternion Response

Case 3: *With Saturation Function*

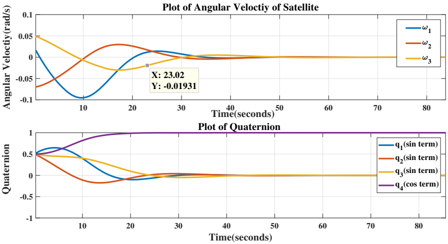

Figure 52: Angular Velocity & Quaternion Response

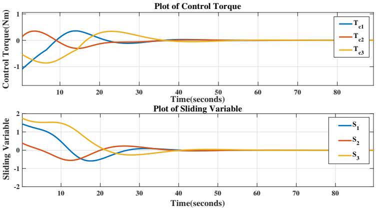

Figure 53: Torque & Sliding Variable Response

Case 4: *With Torque Saturation*

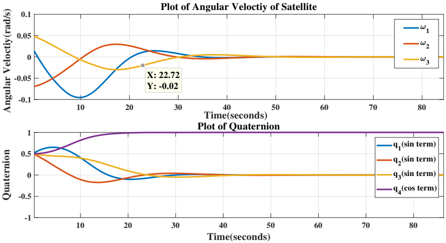

Figure 54: Angular Velocity & Quaternion Response

Case 4: *With Torque Saturation*

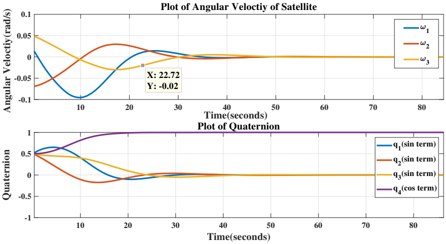

Figure 54: Angular Velocity & Quaternion Response

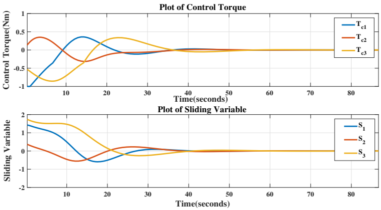

Figure 55: Torque & Sliding Variable Response

Case 5: *With Perturbation*

Both 40% & 60% perturbation applied. Fig. shows response for 60% perturbation

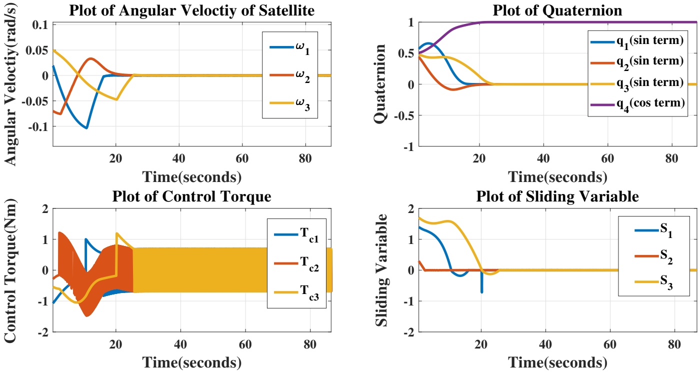

Figure 56: Response in presence of 60 % perturbation

Observation

- In case of PD^{β}-SMC law, following things can be observed:
- System achieves its desired angular velocity & quaternion with T_s=22.98 sec
- Disturbance effect is rejected
- Chattering phenomena is visible for PD^{β}-SMC, which gets removed using saturation function
- With torque saturation, control torque is limited to ± 1 N-m but T_s increases to 22.72 seconds

PD^{β}-SMC with Modified Reaching Law

PD^{β}-SMC with Modified Reaching Law

Case 1: *Without Disturbance*

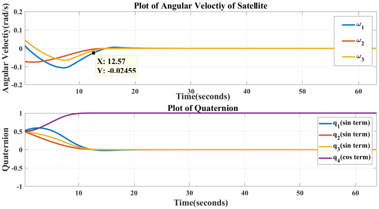

Figure 57: Angular Velocity & Quaternion Response

PD^{β}-SMC with Modified Reaching Law

Case 1: *Without Disturbance*

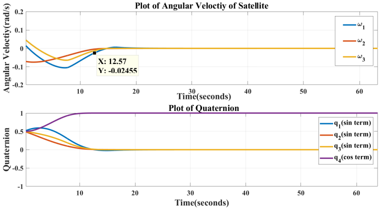

Figure 57: Angular Velocity & Quaternion Response

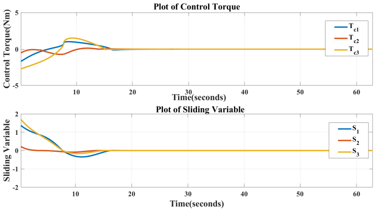

Figure 58: Torque & Sliding Variable Response

Case 2: *With Disturbance*: between 60 to 120 sec

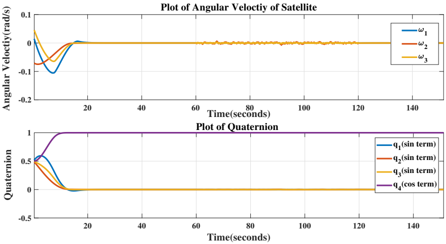

Figure 59: Angular Velocity & Quaternion Response

Case 2: *With Disturbance*: between 60 to 120 sec

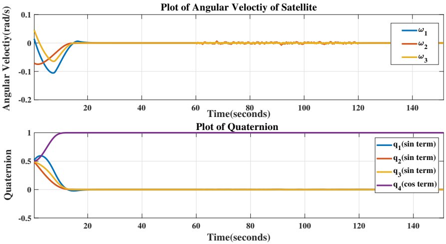

Figure 59: Angular Velocity & Quaternion Response

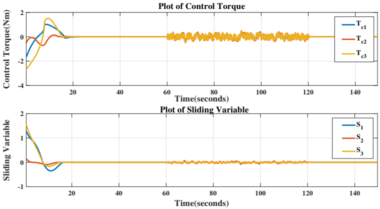

Figure 60: Torque & Sliding Variable Response

Case 3: *With Torque Saturation*

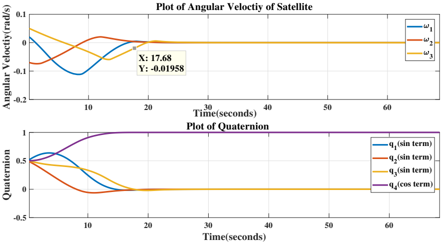

Figure 61: Angular Velocity & Quaternion Response

Case 3: *With Torque Saturation*

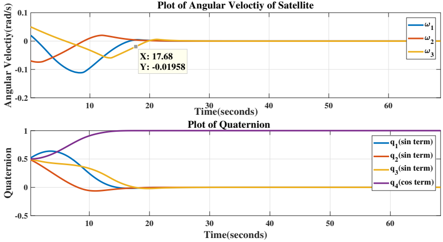

Figure 61: Angular Velocity & Quaternion Response

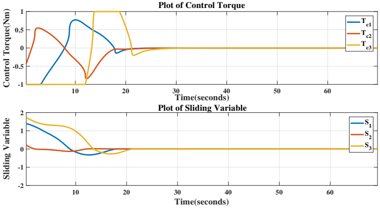

Figure 62: Torque & Sliding Variable Response

Observation

- Following observations can be mentioned:
- System achieves desired responses with T_s=12.57 sec
- Disturbance effect is rejected
- Chattering effect is absent
- Limiting the maximum control torque, T_s raises to 17.68 seconds

Performance Analysis

- Following performance indices are considered for performance analysis:

1. $ITAE_\omega$ (Integral Time absolute error)
2. Control Effort
3. ISE_ω (Integral Square Error)
4. $ITAE_q$
5. Reaching Time T_{reach}

Comparison of Different Controllers

Controller Name	Performance Index				
	$ITAE_\omega$	Control Effort	ISE_ω	$ITAE_q$	Reaching Time (T_{reach}, in seconds)
Linear Controllers					
PD	7.582	403.1	0.1382	22.657	Not Applicable
PD^β	18.86	455.7	0.1381	102.81	Not Applicable
Non Linear Controllers					
PD -SMC	21.24	314.4	0.1082	137.27	23.74
PD-SMC modified	11.64	39.18	0.1159	43.639	15.93
PD^β SMC	20.96	284.8	0.1216	120.09	23.315
PD^β SMC modified	12.230	42.53	0.1214	43.829	15.8

Table 3: Comparison of Performance of Different Controllers

Two performance indices are analyzed here under modifications of control signal:**Control effort, Reaching Time**

Controller	Control Effort		
	Normal Condition	With Saturation Function	With Control Input Saturation
Linear Controllers			
PD	403.1	Not Applicable	40.03
PD^{β}	455.7	Not Applicable	41.36
Non Linear Controllers			
PD-SMC	314.4	18.37	17.09
PD-SMC modified	39.18	Not Applicable	27.56
PD^{β}-SMC	284.8	14.19	14.06
PD^{β}-SMC modified	42.53	Not Applicable	29.04

Table 4: Effect of Control Law Modification on Control Effort

Controller	Reaching Time (T_{reach})		
	Normal Condition	With Saturation Function	With Control Input Saturation
PD-SMC	23.74	44	44.78
PD-SMC modified	15.93	Not Applicable	21.11
PD^{β}-SMC	23.315	54.5	54.607
PD^{β}-SMC modified	15.8	Not Applicable	20.756

Table 5: Effect of Control Law Modification on Reaching Time

Conclusion

Among linear controllers,

- PD law has minimum $ITAE_\omega, ITAE_q$, control effort
- PD^β law has minimum ISE_ω

Among non linear controllers,

- modified PD-SMC has the minimum $ITAE_\omega, ITAE_q$ & control effort index
- PD-SMC has minimum ISE_ω
- T_{reach} is minimum for modified PD^β-SMC, most robust among the controllers concerned

Future Scope

- Design of Terminal Slding Mode Controller to have finite time convergence
- Design of $(PD)^{\beta}$ controller, and compare among PD^{β} & $(PD)^{\beta}$
- Adaptive law design for more robustness
- Controller Design to suppress the effect of disturbance (modelled using real time data).

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Advanced Control Algorithms For Satellite Attitude Maneuver

A project report submitted
in partial fulfillment for the award of the degree of

Bachelor of Technology

in

Electronics & Communication Engineering (Avionics)

by

Debajyoti Chakrabarti

Department of Department of Avionics
Indian Institute of Space Science and Technology
Thiruvananthapuram, India

May, 2019

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